Problem 1.18

Determine the mean square of the rectangular pulse shown in Fig. P1.18 for $k = 0.10$. If the amplitude is A, what would an rms voltmeter read?

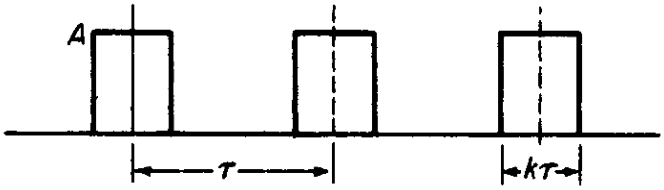

FIGURE P1.18.

Solution

The mean square of a wave $x(t)$ is defined as

$$\overline{x^2} = \frac{\int x^2\,dt}{\int dt}.$$

The wave repeats itself every τ seconds and thus has period τ. Its first cycle can be represented piecewise as

$$x(t) = \begin{cases} A & 0 < t < \frac{k\tau}{2} \\ 0 & \frac{k\tau}{2} < t < \tau - \frac{k\tau}{2} \\ A & \tau - \frac{k\tau}{2} < t < \tau \end{cases}.$$

The mean square of the whole wave can be found by integrating over one cycle.

$$\begin{aligned} \overline{x^2} &= \frac{\displaystyle\int_0^{\frac{k\tau}{2}} (A)^2\,dt + \int_{\frac{k\tau}{2}}^{\tau-\frac{k\tau}{2}} (0)^2\,dt + \int_{\tau-\frac{k\tau}{2}}^{\tau} (A)^2\,dt}{\displaystyle\int_0^{\tau} dt} \\ &= \frac{A^2\left(\frac{k\tau}{2}\right) + A^2\left(\frac{k\tau}{2}\right)}{\tau} \\ &= \frac{A^2 k\tau}{\tau} \\ &= A^2 k. \end{aligned}$$

If $k = 0.10$, the mean square of the wave is

$$\overline{x^2} = 0.10A^2.$$

An rms voltmeter would display the square root of the mean square. Therefore,

$$V_{\text{rms}} = \sqrt{0.10}A \approx 0.3162A.$$